# Higher Order Thinking Skills (HOTS) for class X, Mathematics

## Number system

1. Show that square of any positive odd integer is of the form 8q+1 for some integer q.

2. Find whether the decimal representation (expansion) of the numbers (i) 51/120 (ii) 637/7280 terminating or non-terminating

3. The decimal expansion of following numbers, will terminate after how many places

   (i) $\frac{47}{2^3 5^2}$ (ii) $\frac{359}{2 \times 54}$

4. Prove that $3 + 2\sqrt{5}$ is an irrational number.

## Polynomials

5. Find all zeroes of the polynomials $x^4 - 5x^3 - 9x^2 + 15x + 18$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$

6. Find all zeroes of the polynomial $x^4 - 4x^3 - 2x^2 + 12x - 3$ if two of its zeroes are $2 \pm \sqrt{3}$

7. Obtain all zeroes of the polynomial $5x^3 - 15x^2 - 3x + 9$, if two of its zeroes are $\sqrt{\frac{3}{5}}$ and $-\sqrt{\frac{3}{5}}$

## Linear Equations

8. Solve the following pair of equations:

   $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$, $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

9. A chemist has one solution which is 50% acid and a second solution which is 25% acid. How much of each should be mixed to make 10 litres of a 40% acid solution.

10. Check graphically whether the pair of equations

    $x + 3y = 6$, $3x + 9y = 12$

    is consistent. Also find the points where these line intersect x-axis.

11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Sarita paid Rs. 27 for a book kept for seven days

while Sunita paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

**Quadratic Equations**

12. Find the value of k for which the roots of the quadratic equation $kx^2-10x+5=0$ are equal.

13. Find the value of p for which the roots of the quadratic equation $3x^2-px+3=0$ are real where $p > 0$

14. Sum of the areas of two squares is 468 $m^2$. If the difference of their perimeters is 24 m, find the sides of two squares.

15. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr. more, it would have taken 1 hour less for the same journey. Find the speed of the train.

**Arithmetic Progressions**

16. The 19 th term of an AP is equal to 3 times its 6 th term. If the 9 th term of the AP is 19, find the AP.

17. Find the sum of first 19 terms of an AP whose 8 th term is 41 and 13 th term is 61.

18. Find the sum of all three digit numbers which are divisible by 9.

19. For what value of K, 2K-7, K+5 and 3K+2 are three consecutive terms of an AP.

20. If the sum of first n terms of an AP is given by $3n^2 + 5n$, find the common difference of the AP.

21. The sum of first three terms of an AP is 15. If the sum of their squares is 93, find the AP.

**Trigonometry**

22. Prove that $\sec^4\theta(1-\sin^4\theta) - 2\tan^2\theta = 1$

23. If $\sin\theta + \sin^2\theta = 1$ prove that $\cos^2\theta + \cos^4\theta = 1$

24. If sin(A+B) = 1 and cos(A-B)=1, $0 < A+B \leq 90^\circ$, $A \geq B$, find A and B

25. Evaluate : $\dfrac{\tan 2^\circ \tan 3^\circ \ldots\ldots \tan 45^\circ \ldots\ldots \tan 87^\circ \tan 88^\circ}{(3\sin^2 43^\circ + 3\sin^2 47^\circ) - 2(\sec^2 39^\circ - \cot^2 51^\circ)}$

26. Prove that $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{1 + \cos A}{\sin A}$

**Coordinate Geometry**

27. Show that the points (-3,5), (3,1), (0,3) and (-1,-4) do not form a quadrilateral.

28. Find the Coordinates of Point P on AB such that $\dfrac{PA}{PB} = \dfrac{3}{4}$ where A(3,1) and B(-2,5)

29. Points P,Q,R and S in that order divides the line segment joining points A(2,5) and B(7,-5) in five equal parts. Find the coordinates of P,Q,R and S.

30. The vertices of a triangle are (1,K), (4,-3) and (-9,7). If the area of triangle is 15 sq. units, then find the value(s) of K.

**Geometry**

31. In Fig.1. ABC is an isosceles triangle with AB=AC. P is a point on the side BC such that $PM \perp AB$ and $PN \perp AC$. Prove that BM x NP = CN x MP

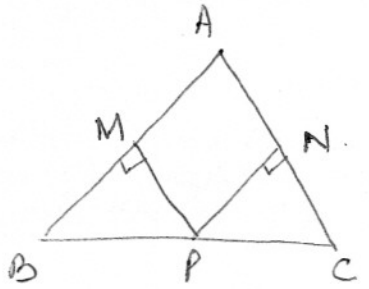


fig 1.

32. In Fig.2, $\angle ACB = 90^\circ$ and $CD \perp AB$

Prove that $\dfrac{CB^2}{CA^2} = \dfrac{BD}{AD}$

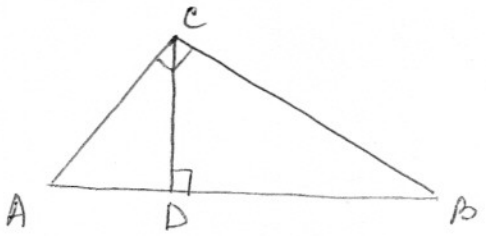


fig 2

33. In a $\triangle$ ABC, D is the mid point of side BC and $AE \perp BC$. If $AC > AB$, Show that $AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC \cdot DE$

34. In Fig. 3, XY and X'Y' are two parallel tangents to a circle with Centre O and another tangent AB touching at C, intersecting XY at A and x y at B. Prove that AOB=$90^0$

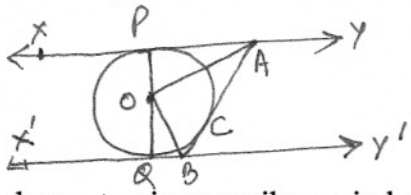


fig 3

35. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of length 8 cm and 6 cm respectively. (see Fig.4). Find the sides AB and AC.

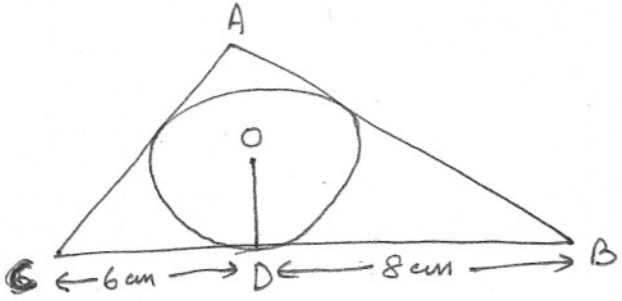


fig 4.

36. A round table cover has six equal designs as shown in figure on the side (shaded one). If the radius of the cover is 28 cm, find the cost of making the design at the rate of Rs. 0.35 per $cm^2$ (Use $\sqrt{3} = 1.7$ ).

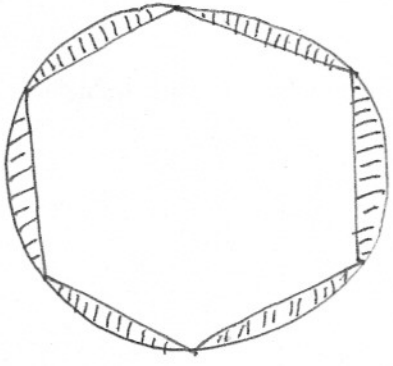

fig 5

37. Two circles touch each other externally and the sum of their areas is 52 $cm^2$. If the distance between the centres of two circles is 10 cm, find the radii of the two circles.

38. Find the area of the shaded region in the figure on the side where ABCD is a square of side 14 cm (Use $\pi = \frac{22}{7}$).

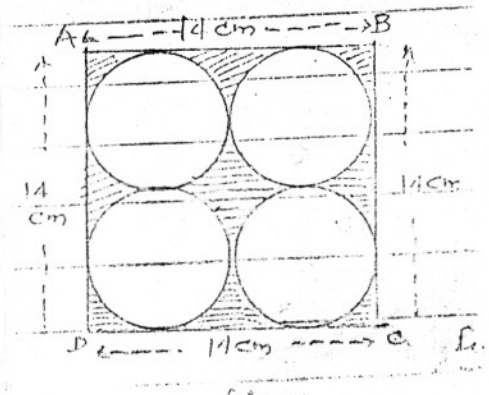


fig. 6

39. In the given figure, from each corner of a square ABCD, of side 4 cm, quadrant of a circle of radius 1 cm each is cut and a circle of radius 1 cm is cut from the centre. Find the area of the shaded region.

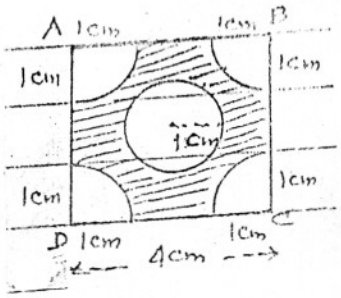


fig. 7

40. Find the area of the shaded region which contains two semi-circles and a rectangle of breadth 1 cm.

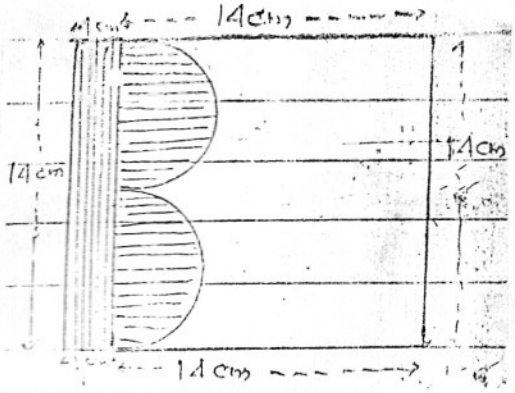


fig 8.

41. A metallic cylinder has a radius 3 cm and height 5 cm. It is made of metal A. To reduce its weight a conical hole is drilled in the cylinder (as shown in the figure)

and it is completely filled with a lighter metal B. The conical hole has a radius of 3/2 cm and its depth is 8/9 cm. Calculate the ratio of the volume of the metal A to the volume of metal B in the solid.

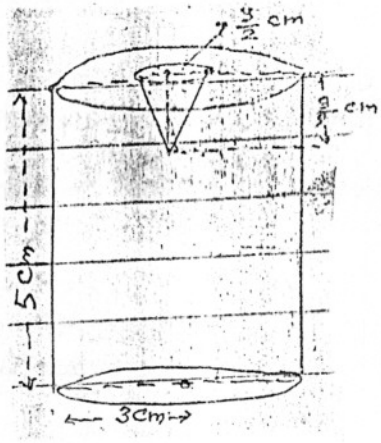


fig 9

42. Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the bigger sphere.

43. A spherical copper shell of external diameter 18 cm is melted and recaste into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm. Find the inner diameter of the shell.

44. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm X 10 cm X 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

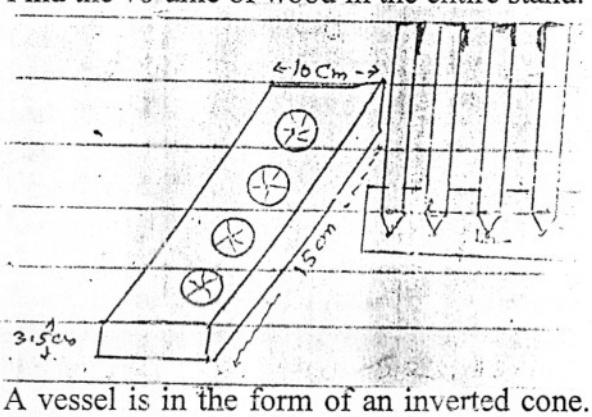


fig 10

45. A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped.

46. A container, shaped like a circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 6 cm and diameter 3 cm having a hemispherical shape on the top of the same radius of base as that of cone. Find the number of such cones that can be filled with ice-cream.

47. A container, open at the top and made of metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of Rs. 20 per litre. Also, find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 $cm^2$ (Use $\pi = 3.14$)

48. A metallic right circular cone 20 cm high and whose vertical angle is $60^0$ is cut into two parts at the middle of its height by a plane parallel to the base. If the frustum so obtained is melted and drawn into a wire of diameter 1 cm, find the length of the wire.

49. The median of the following data is 525. Find the values of x and y if the total frequency is 100:

| Classes | 0-100 | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 | 600-700 | 700-800 | 800-900 | 900-1000 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 2 | 5 | X | 12 | 17 | 20 | Y | 9 | 7 | 4 |

50. During the medical check up of 35 students of a class, their weights were recorded as follows-

| **Weight (in kg)** | **Number of students** |
|---|---|
| Less than 38 | 0 |
| Less than 40 | 3 |
| Less than 42 | 5 |
| Less than 44 | 9 |
| Less than 46 | 14 |
| Less than 48 | 28 |
| Less than 50 | 32 |
| Less than 52 | 35 |

Draw a less than type ogive for the above data. Hence obtain the median weight from the graph and verify the result by using the formula.

Class IX

1. To find the percentage of students in a group of students who write faster with their left hand / right hand.

2. To help the students establish interesting mathematical relationships by measuring some parts of the body.

List of projects given as examples

1. **Observing interesting patterns in cricket match.**
   Comparison of the performance of two teams in a one-day international cricket match.

2. **Design a crossword puzzle with mathematical terms**
   To review mathematics vocabulary, to give the opportunity for creative expressions in designing puzzles, to act as a means of monitoring the study of a given unit and to give recreation.

3. **A measuring task**
   To investigate your local athletics track to see whether it is marked fairly for runners who start on different lines.

4. **Project in history of mathematics**
   i. Study various aspects of Pythagoras theorem.
   ii. Investigation of various historical aspects of number $\pi$.

Suggested list Of Projects

**P1 Cricket**

Collect data on runs scored in each over for a one-day international (ODI) cricket match and obtain frequency distribution between runs and overs. Do this for both the teams and also for the first 25 and the remaining overs of the match. Observe any interesting features of the match. Compare it with similar analysis for a few other ODI's.

**P2 Age profile in your neighbourhood**

Survey any 30 households in your locality and collect data on the age of the persons. Determine the age profile (number of persons Vs age) for men and women. Report any significant observation from the data.

**P3 Educational Background in your neighbourhood**

Survey any 30 households in your locality and collect data on the educational background of the persons. Obtain significant observations from your data.

**P4 Number of Children in a family in your neighbourhood**

Survey any 50 households in your locality and collect data on the number of children (male and female) in each family. Report any significant observation.

**P5 Making of Platonic solids**

Obtain and construct the nets of five platonic solids. Make these solids and observe the properties (number of faces, edges and vertices) of the solids. Try to find out, why there are only five platonic solid. (Try taking regular hexagon)

**P6 History of Mathematics**

Refer history of mathematics sources from your library or Internet and prepare a poster or a document on any topic of your interest. The students can choose several topics from history of mathematics, for doing a project. For instance the topic can be about an Indian mathematician or the concept of zero in various ancient civilizations.

**P7 Mathematics line designs**

Using strings obtain interesting designs and patterns. Use threads and shapes made by cardboard, try to make designs on it by making slits on the cardboard. Observe different patterns on it.

**P8 Computer project**

Using a spreadsheet programme on a PC obtain the graph of the equation $ax^2 + bx + c = 0$ for a different values of a, b and c and note the interesting features and patterns. Interested students can also try for quadratic equations.

Class X

# Projects

**Project 1: Efficiency in Packing**

To investigate the efficiency of packing of objects of different shapes in a cuboid box. *(Efficiency is the percentage of box space occupied by the objects.)*

**Project 2: Geometry in real life**

In this project we try to find situations in daily life where geometrical notions can be effectively used. In particular, the student discovers situations in which properties of similar triangles learnt in the classroom are useful.

**Project 3: Experiments on Probability**

To appreciate that finding probability through experiment is different from finding proability by calculation. Students become sesitive towards the fact that if they increase the number of observaions, probabilty found through experiment approaches the calculated probability.

**Project 4: Displacement and rotation of a geometrical figure**

To study the distance between different points of a geometrical figure when it is displaced and / or rotated. Enhances familiarity with co-ordinate geometry.

**Project 5: Frequency of letters/ words in a language**

Analysis of a language text using graphical and pie chart techniques.

# Group Activities

**Group activity 1**

**Fourth order Magic Dance**

The interplay of mathematics and art can be very appealing. This activity makes an attempt to present a versatile form of the fourth order magic square through a dance.

**Group activity 2**

**Live Lattice**

Live lattice is a lattice formed by students placed in square or rectangular formation.

# Suggested Projects

**Project 1**

Mathematical designs and patterns using arithmetic progression

**Project 2**

Early history of Mathematics

**Project 3**

Analysis of test results and interpretation